Image processing
"Histogram-based operators"

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Lecture available at http://mathieu.delalandre.free.fr/teachings/image.html

Histogram-based operators

1. Fundamentals of histogram-based operators
2. Some histogram-based operators
   - 2.1. Image characterization
   - 2.2. Optimum thresholding
   - 2.3. Histogram equalization
   - 2.4. Histogram-based distances
3. Further investigations

Fundamentals of histogram-based operators (1)

The histogram of a digital image is a representation of its intensity distribution such as

The image

I(i,j) = v	is a discrete function
i,j	the coordinates of a pixel
$i \in [0, N[$ and $j \in [0, M[$	
v	is the pixel intensity value with $0 \leq v \leq L$
M×N	is the size of the array (in pixels)

The histogram

$h(k) = n_k$	is a discrete function
k	the intensity value
$k \in [0, L]$	is the intensity level range
n_k	is the number of pixels in the image of intensity k

$$\sum_{k=0}^{L} h(k) = N \times M$$

e.g.

Raster with
N=3
M=4
N×M=12
q = 3
$0 \leq I(i,j) \leq 7$

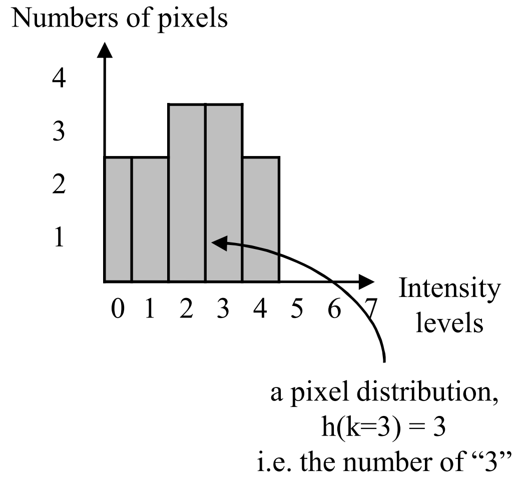
1	2	1	4
2	0	3	3
3	2	0	4

a pixel
I(1,2) = 3

→

Histogram with
$k \in [0, 7]$

$$\sum_{k=0}^{7} h(k) = 12$$



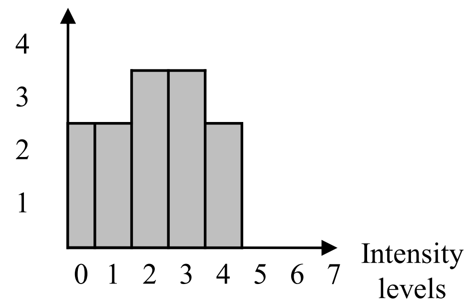
Fundamentals of histogram-based operators (2)

Raster with
N=3
M=4
N×M=12
q = 3
$0 \le I(i,j) \le 7$

1	2	1	4
2	0	3	3
3	2	0	4

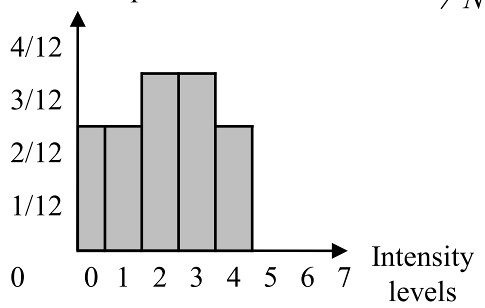
The basic image histogram is

Numbers of pixels $h(k) = n_k$



The normalized image histogram is

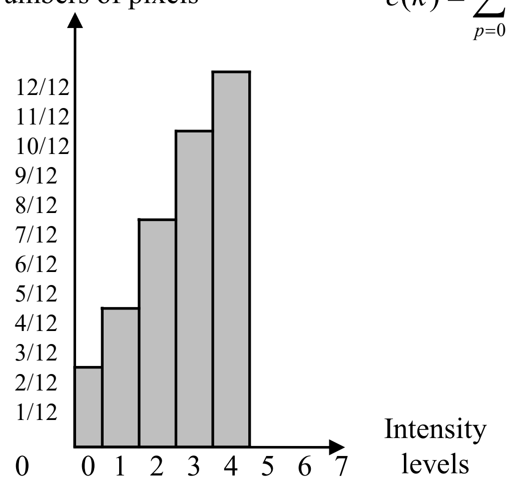
Numbers of pixels $p(k) = h(k) / N \times M$



p(k) defines the probability to get the value k in the image I

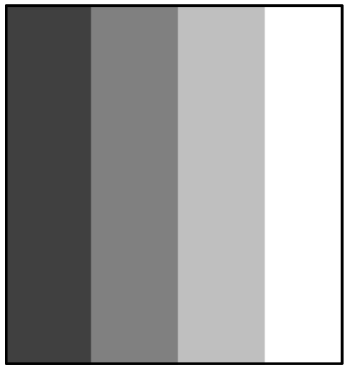
The accumulated image histogram is

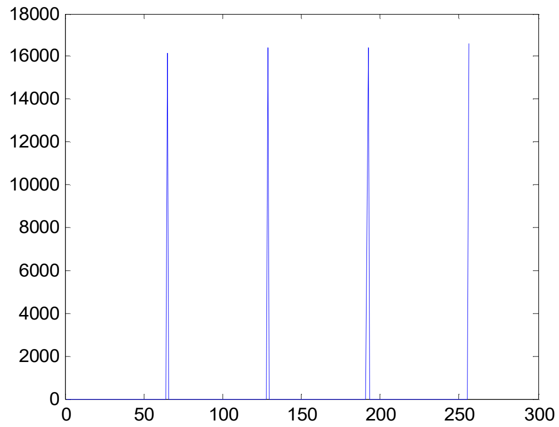
Numbers of pixels $c(k) = \sum_{p=0}^{k} p(p)$



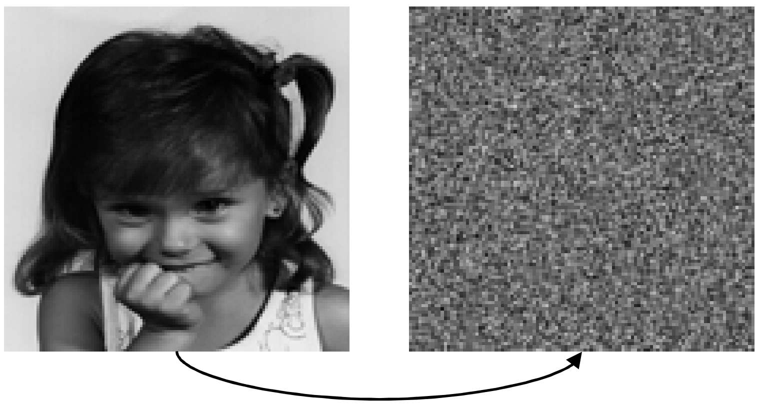
c(k) defines the probability to get the value less or equal to k in the image I

Fundamentals of histogram-based operators (3)

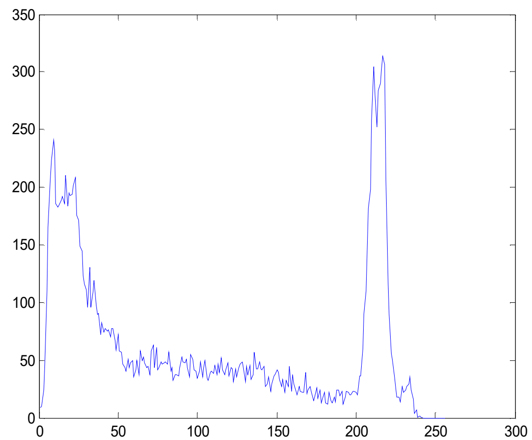




n-colors images result in n-bins histograms



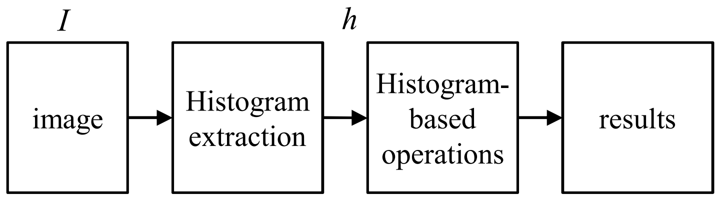
Random pixel permutation



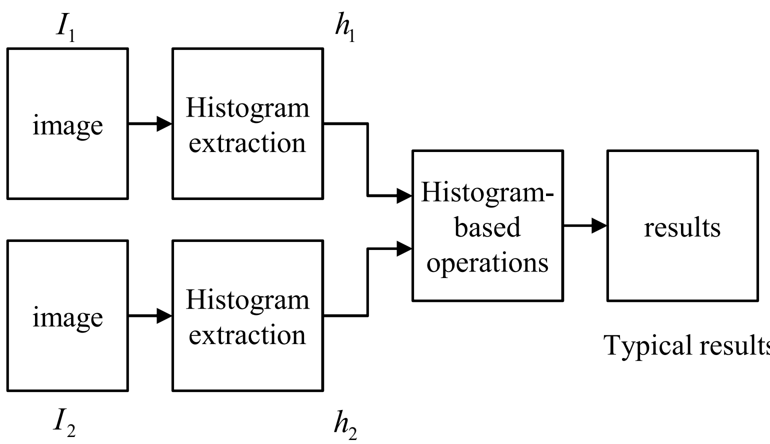
histograms are global representations excluding image topology

Fundamentals of histogram-based operators (4)

Histogram-based operations include any statistical processes of intensity distribution. They could be based on single or multiple entries.

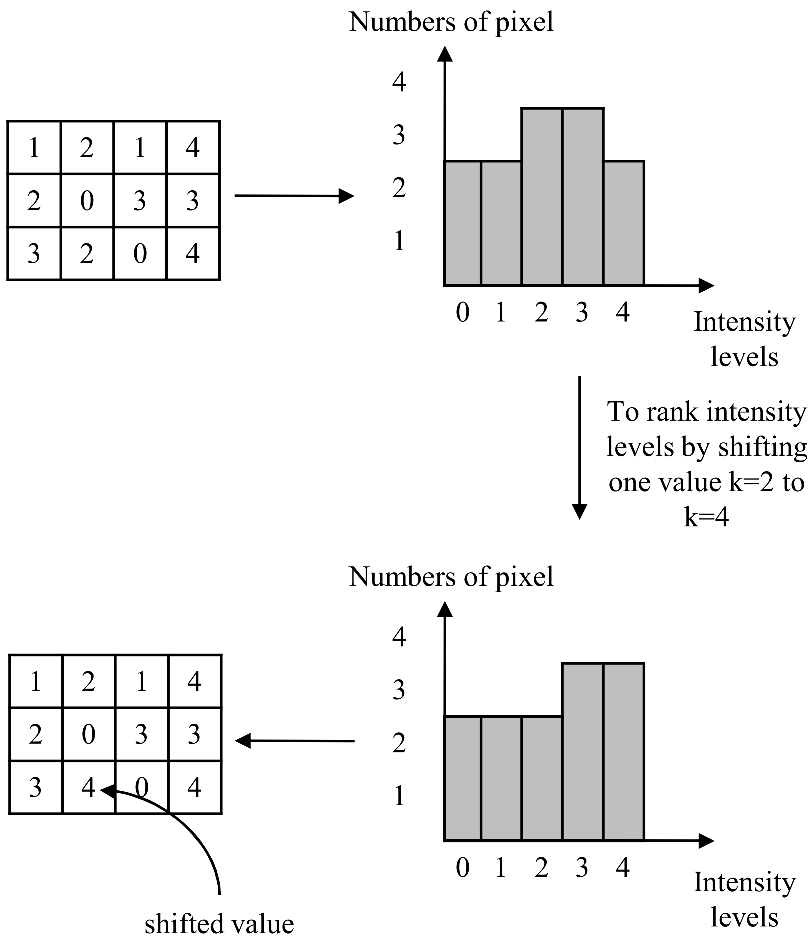


Typical results include
- an image
- some features



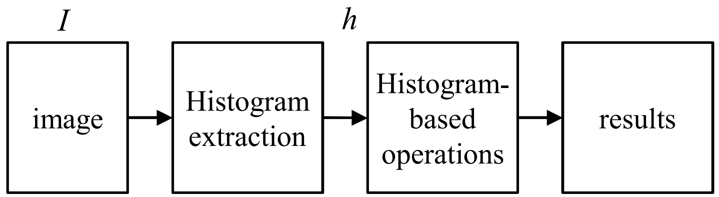
Typical results are distances

e.g.

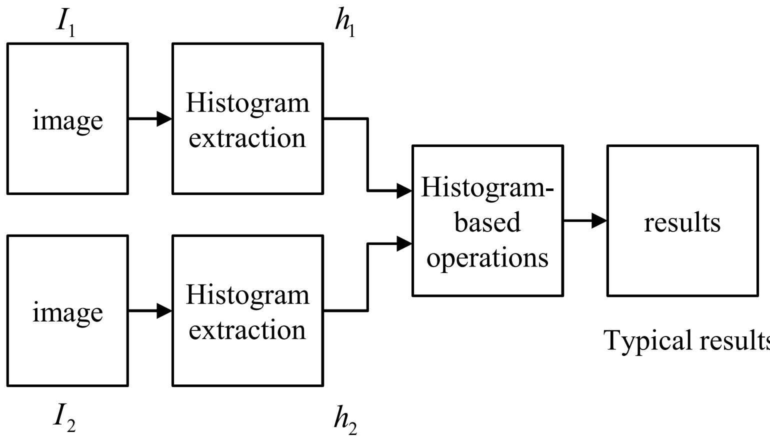


Fundamentals of histogram-based operators (5)

Histogram-based operations include any statistical processes of intensity distribution. They could be based on single or multiple entries.



Typical results include
- an image
- some features



Typical results are distances

Histogram-based operations are related to

- ✓ image characterization (or features extraction),
- ✓ automatic thresholding,
- ✓ image enhancement (histogram equalization),
- ✓ image matching (histogram-based distances),
- ✓ etc.

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   2.4. Histogram-based distances
3. Further investigations

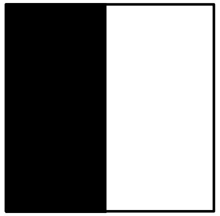
Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

Image characterization
"Mean and standard deviation"

Characterization with mean and standard deviation

	From raster	From histogram	From normalized histogram
Mean (m)	$\frac{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1} I(i,j)}{N \times M}$	$\frac{\sum_{k=0}^{L} k \times h(k)}{N \times M}$	$\sum_{k=0}^{L} k \times p(k)$
Standard deviation (σ)	$\frac{\sqrt{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\left(I(i,j)-m\right)^2}}{N \times M}$	$\frac{\sqrt{\sum_{k=0}^{L}\left(k-m\right)^2 \times h(k)}}{N \times M}$	$\sqrt{\sum_{k=0}^{L}\left(k-m\right)^2 \times p(k)}$

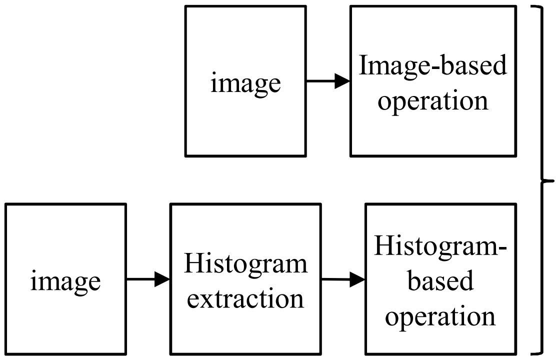
$$m = \frac{L}{2}$$
$$\sigma = 0$$



$$m = \frac{1}{2} \times 0 + \frac{1}{2} \times L = \frac{L}{2}$$
$$\sigma = \sqrt{2 \times \frac{1}{2} \times \left(\frac{L}{2}\right)^2} = \frac{L}{2}$$

Rq. Standard deviation is also defined as the squared root of the variance $\sigma = \sqrt{v}$

Complexity comparison of raster vs. histogram-based operations



	access / increment operations	arithmetic operations			
		add/ subtract	multiply	square	divide
Mean raster	N×M	N×M	1	0	1
Mean histogram	2×N×M	2^q	2^q	0	1
STD raster	N×M	2×N×M	1	N×M	1
STD histogram	2×N×M	2×2^q	2^q	2^q	1

with $N \times M >> 2^q$
and $O(access) << O(arithmetic)$

the histogram-based operations are most efficient.

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

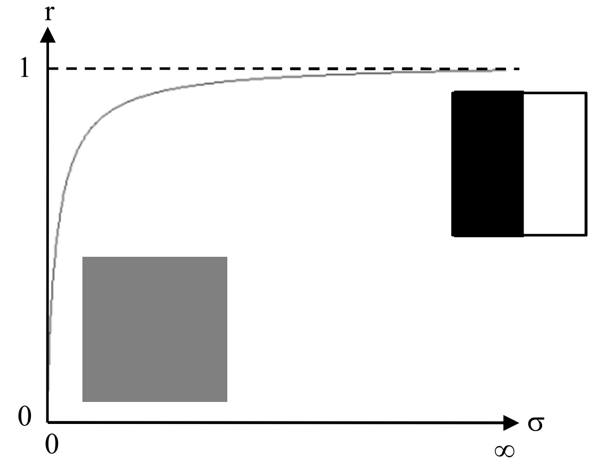
Image characterization
"Contrast" (1)

Characterization with contrast

	From standard deviation	From standard deviation (normalized)
Contrast (r)	$r = 1 - \frac{1}{1+\sigma^2}$	$r = 1 - \frac{1}{1+\frac{\sigma^2}{L^2}}$

Contrast is the difference in visual properties that makes objects in an image distinguishable from other objects and the background. There are many possible definitions of contrast, the one above is the variance-based.

Due to the large values meet for variance, a normalization of variance with the number of square level intensity could improve the global curvature function.



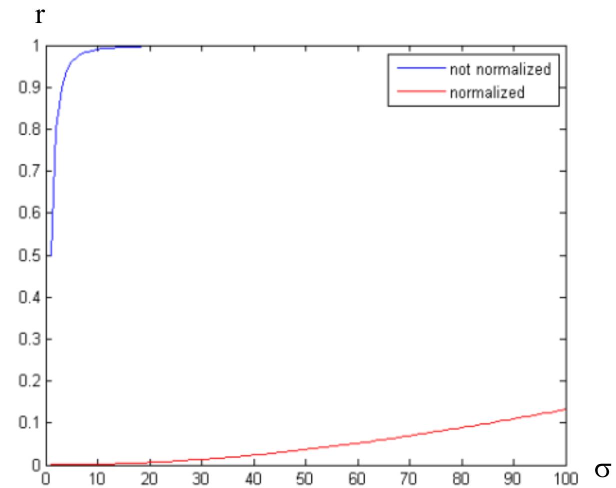


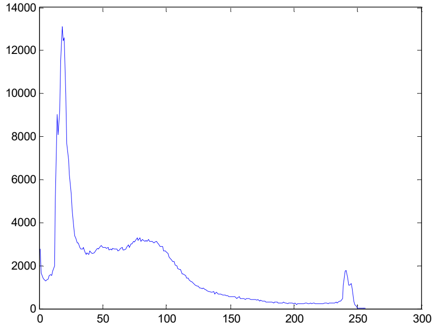
Image characterization
"Contrast" (2)

Characterization with contrast

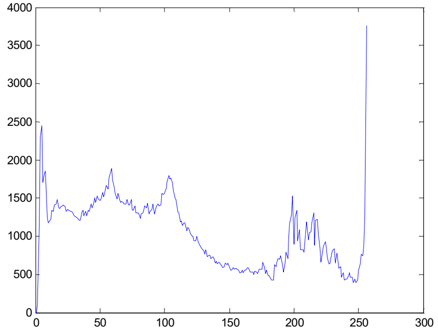
	From standard deviation	**From standard deviation (normalized)**
Contrast (r)	$r = 1 - \frac{1}{1+\sigma^2}$	$r = 1 - \frac{1}{1+\frac{\sigma^2}{L^2}}$

Contrast is the difference in visual properties that makes objects in an image distinguishable from other objects and the background. There are many possible definitions of contrast, the one above is the variance-based.

Due to the large values meet for variance, a normalization of variance with the number of square level intensity could improve the global curvature function.



Standard deviation σ	2 108
Contrast (r)	1
Normalized contrast (r)	0,98



Standard deviation σ	457
Contrast (r)	1
Normalized contrast (r)	0,76

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
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Image characterization
"Statistical moments" (1)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment (μ_n)	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

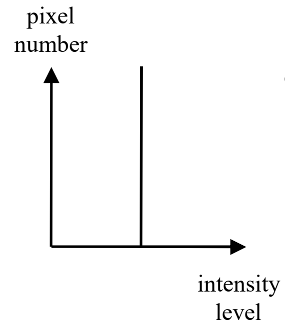
n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0th moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1st moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2sd moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3rd moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4th moment measures the flatness

e.g.

k	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
h(k)	0	0	0	80	0	0	0	0
p(k)	0	0	0	1	0	0	0	0
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-3	-2	-1	0	1	2	3	4
$(k-m)^2$	9	4	1	0	1	4	9	4
$(k-m)^3$	-27	-8	-1	0	1	8	27	64
$(k-m)^4$	81	16	1	0	1	16	81	256

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 3$$

1	μ_0
0	μ_1
0	μ_2
0	μ_3
0	μ_4



constant image
null variance
null skewness
null flatness

Image characterization “Statistical moments” (2)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment (μ_n)	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

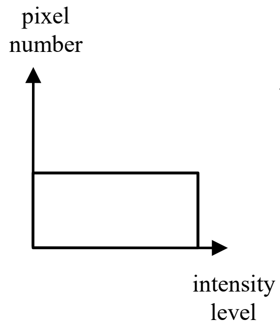
n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0th moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1st moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2sd moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3rd moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4th moment measures the flatness

e.g.

k	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
h(k)	10	10	10	10	10	10	10	10
p(k)	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-3,5	-2,5	-1,5	-0,5	0,5	1,5	2,5	3,5
$(k-m)^2$	12,5	6.25	2.25	0.25	0.25	2.25	6.25	12.25
$(k-m)^3$	-42,8	-15-6	-3.3	-0.12	0.12	3.3	15.6	42.8
$(k-m)^4$	150	39	5	0.06	0.06	5	39	150

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 3.5$$

1	μ_0
0	μ_1
5.25	μ_2
0	μ_3
48.6	μ_4



gradient image
strong variance
null skewness
strong flatness

Image characterization “Statistical moments” (3)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment (μ_n)	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

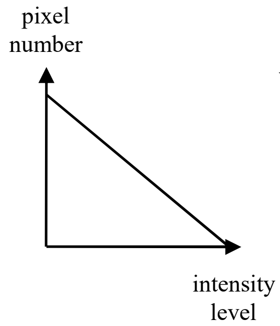
e.g.

k	0	1	2	3	4	5	6	7
h(k)	70	60	50	40	30	20	10	00
p(k)	7/28	6/28	5/28	4/28	3/28	2/28	1/28	0
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-2	-1	0	1	2	3	4	4
$(k-m)^2$	4	1	0	1	4	9	16	25
$(k-m)^3$	-8	-1	0	1	8	27	64	125
$(k-m)^4$	16	1	0	1	16	81	256	625

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 2$$

1	μ_0
0	μ_1
3	μ_2
3	μ_3
21	μ_4

n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0th moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1st moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2sd moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3rd moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4th moment measures the flatness



weighted gradient image
medium variance
strong skewness
medium flatness

Image characterization “Statistical moments” (4)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment (μ_n)	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

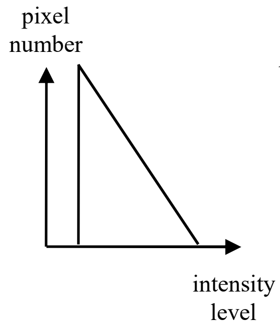
e.g.

n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0th moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1st moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2sd moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3rd moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4th moment measures the flatness

k	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
h(k)	0	0	0	40	30	20	10	00
p(k)	0	0	0	4/10	3/10	2/10	1/10	0
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-4	-3	-2	-1	0	1	2	3
$(k-m)^2$	16	9	4	1	0	1	4	9
$(k-m)^3$	-64	-27	-8	-1	0	1	8	27
$(k-m)^4$	256	81	16	1	0	1	16	81

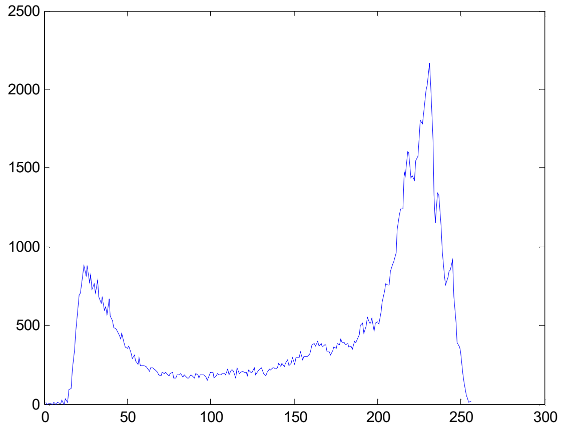
$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 4$$

1	μ_0
0	μ_1
1	μ_2
0.6	μ_3
2.2	μ_4

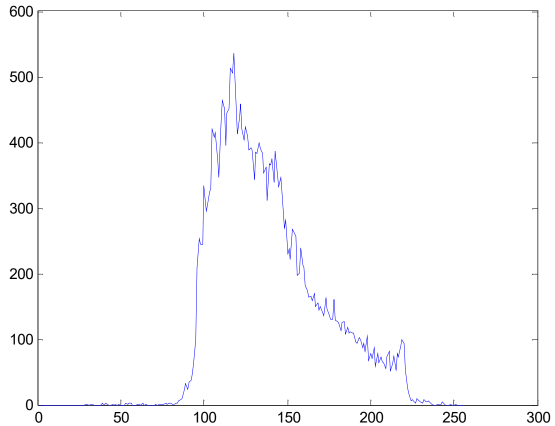


band weighted gradient image
- weak variance
- weak skewness
- weak flatness

Image characterization
"Statistical moments" (5)



n	value	scale
μ_0	1	10^0
μ_1	0	10^0
μ_2	5.86	10^3
μ_3	-32.07	10^3
μ_4	6720.60	10^3



n	value	scale
μ_0	1	10^0
μ_1	0	10^0
μ_2	0,96	10^3
μ_3	23,05	10^3
μ_4	2764.90	10^3

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu’s method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

Image characterization
"Entropy" (1)

Characterization with Entropy

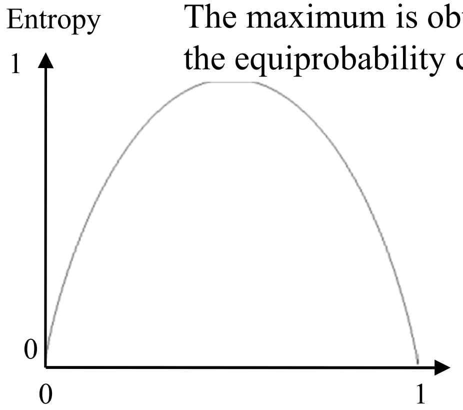
	From histogram	From normalized histogram
Entropy (e)	$-\sum_{k=0}^{2^q-1} \frac{h(k)\times\log_b\left(h(k)/M\times N\right)}{M\times N}$	$-\sum_{k=0}^{2^q-1} p(k)\times\log_b\left(p(k)\right)$

b is the base of the logarithm, common value is 2

Entropy measures the randomness of the image.

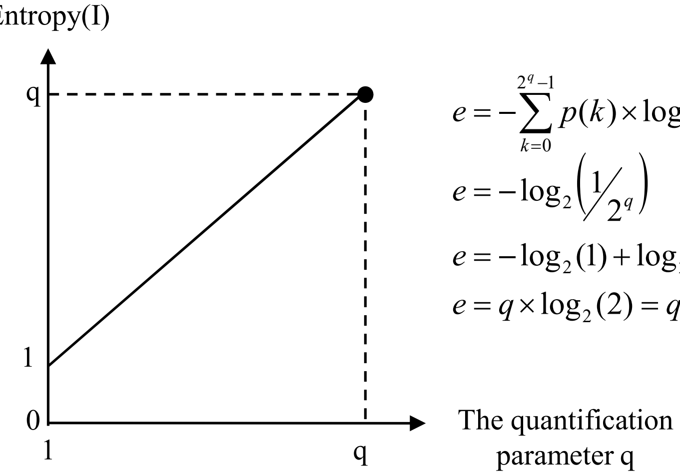
Entropy of two variables (x,y) with b=2 ($\log_2$) and $x = p(0) \quad y = p(1) = 1 - x$

The maximum is obtained for the equiprobability case.



$$e = -\left(x\times\log_2(x) + (1-x)\times\log_2(1-x)\right)$$

Max Entropy values (equiprobability) of n variables with b=2 ($\log_2$)



$$e = -\sum_{k=0}^{2^q-1} p(k)\times\log_b\left(p(k)\right)$$

$$e = -\log_2\left(1/2^q\right)$$

$$e = -\log_2(1) + \log_2(2^q)$$

$$e = q\times\log_2(2) = q$$

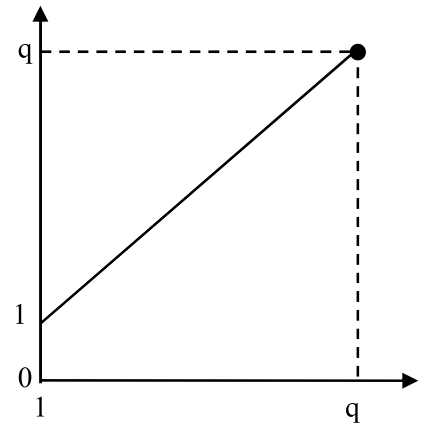
Image characterization “Entropy” (2)

Characterization with Entropy

	From histogram	From normalized histogram
Entropy (e)	$-\sum_{k=0}^{2^q-1} \frac{h(k)\times\log_b\left(h(k)/M\times N\right)}{M\times N}$	$-\sum_{k=0}^{2^q-1} p(k)\times\log_b\left(p(k)\right)$

b is the base of the logarithm, common value is 2

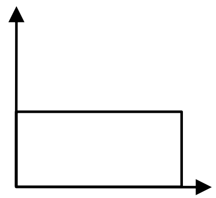
Max Entropy values (equiprobability)



Entropy measures the randomness of the image.

e.g.

k	0	1	2	3	4	5	6	7	res	
h(k)	10	10	10	10	10	10	10	10	80	pixel number
p(k)	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1	sum of probability
$\log_2(p(k))$	-3	-3	-3	-3	-3	-3	-3	-3	3	Entropy (e)



gradient image max entropy

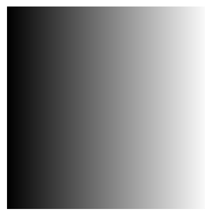


Image characterization “Entropy” (3)

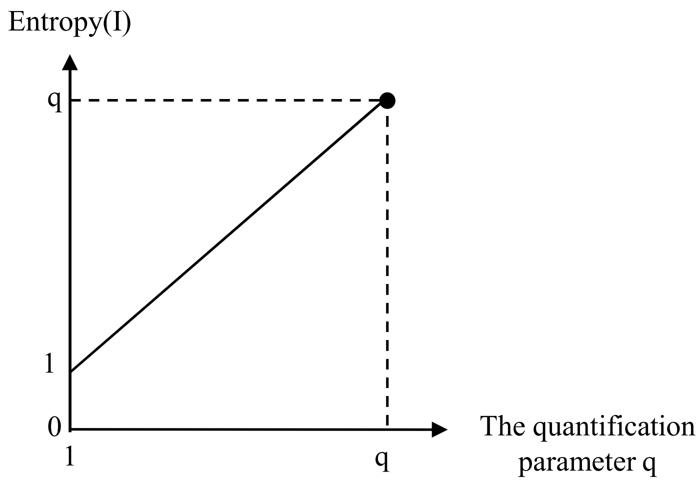
Characterization with Entropy

	From histogram	From normalized histogram
Entropy (e)	$-\sum_{k=0}^{2^q-1} \frac{h(k)\times\log_b\left(h(k)/M\times N\right)}{M\times N}$	$-\sum_{k=0}^{2^q-1} p(k)\times\log_b\left(p(k)\right)$

b is the base of the logarithm, common value is 2

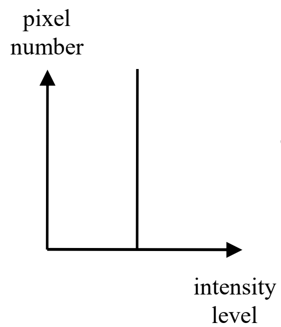
Entropy measures the randomness of the image.

Max Entropy values (equiprobability)



e.g.

k	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**res**	
h(k)				80					80	pixel number
p(k)	0	0	0	1	0	0	0	0	1	sum of probability
$\log_2\left(p(k)\right)$	-∞	-∞	-∞	0	-∞	-∞	-∞	-∞	0	Entropy (e)



constant image
null entropy

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

Image characterization
"Co-occurrence matrix"

equation	description
$n = \sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} g(i,j)$	Co-occurrence number
$p(i,j) = {g(i,j)}/{n}$ $\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} p(i,j) = 1$	Probability estimation of g

A co-occurrence matrix is a distribution that is defined over an image to be the distribution of co-occurring values at a given offset.

an image I
N×M = 6 ×6
q = 3 (i.e. k ∈ [0-7]

0	0	6	4	2	1
4	0	5	0	1	4
7	7	5	7	0	1
3	2	3	4	4	0
7	6	7	6	5	1
6	7	5	1	5	1

The corresponding co-occurrence matrix g using a [1 1] structuring element

k (intensity levels) - j (columns); k (intensity levels) - i (rows)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	1	2				1	1	
1					1	1		
2		1		1				
3			1		1			
4	2		1		1			
5	1	3						1
6					1	1		2
7	1					2	2	1

The corresponding probability estimation p, n equals 30

k (intensity levels) - j (columns); k (intensity levels) - i (rows)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	1/30	2/30				1/30	1/30	
1					1/30	1/30		
2		1/30		1/30				
3			1/30		1/30			
4	2/30		1/30		1/30			
5	1/30	3/30						1/30
6					1/30	1/30		2/30
7	1/30					2/30	/302	1/30

using a [1 1] structuring element, 5 is neighbor of 0, we increase the corresponding g(i,j) element in the co-occurrence matrix

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
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Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

Image characterization
“Uniformity, homogeneity” (1)

Characterization with uniformity and homogeneity

	Equation
Uniformity	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} p(i,j)^2$
Homogeneity	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} \frac{p(i,j)}{1+\lvert i-j \rvert}$

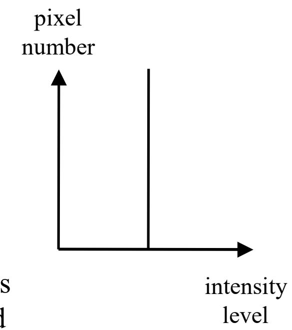
Uniformity (also called Energy) estimates image as a constant.
Homogeneity measures the spatial closeness of the element distribution in g.

e.g.

k (intensity levels) - j (columns); k (intensity levels) - i (rows)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0								
1								
2								
3				1				
4								
5								
6								
7								

	values
Uniformity	$=1^2=1$
Homogeneity	$\frac{1}{1+\lvert 1-1 \rvert}=1$



in a constant image,
the uniformity equals
the homogeneity and
reaches the maximum 1

Image characterization
“Uniformity, homogeneity” (2)

Characterization with uniformity and homogeneity

	Equation
Uniformity	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} p(i,j)^2$
Homogeneity	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} \frac{p(i,j)}{1+\lvert i-j\rvert}$

Uniformity (also called Energy) estimates image as a constant.
Homogeneity measures the spatial closeness of the element distribution in g.

e.g.

k (intensity levels) - j (columns); k (intensity levels) - i (rows)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	1							
1		1						
2			1					
3				1				
4					1			
5						1		
6							1	
7								1

	values
Uniformity	$2^q \times \left(\frac{1}{2^q}\right)^2 = \frac{1}{2^q}$
Homogeneity	$2^q \times \frac{1}{2^q} = 1$

in a gradient image, uniformity is the inverse of the intensity level number, no impact is observed on homogeneity

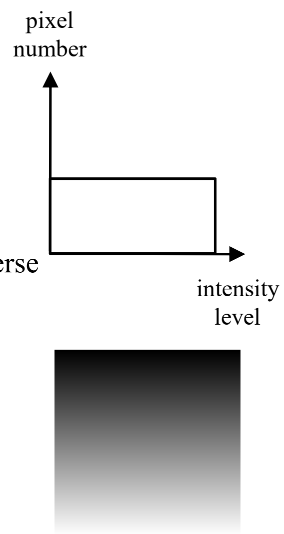


Image characterization
"Uniformity, homogeneity" (3)

Characterization with uniformity and homogeneity

	Equation
Uniformity	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} p(i,j)^2$
Homogeneity	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} \frac{p(i,j)}{1+\lvert i-j \rvert}$

Uniformity (also called Energy) estimates image as a constant.
Homogeneity measures the spatial closeness of the element distribution in g.

e.g.

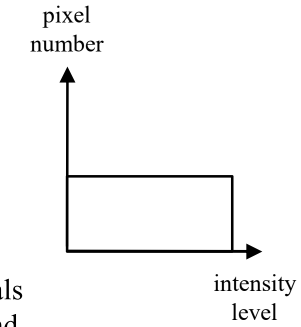
k (intensity levels) - j

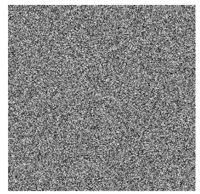
k (intensity levels) - i

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1

	values
Uniformity	$(2^q)^2 \times \left(\frac{1}{(2^q)^2}\right)^2 = \frac{1}{(2^q)^2} \to 0$
Homogeneity	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} \frac{p}{1+\lvert i-j \rvert} = \sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1} \frac{1}{(2^q)^2} \times \frac{1}{1+\lvert i-j \rvert} \to 0$

in a random image,
the uniformity equals
the homogeneity and
reaches the minimum 0





Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
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	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
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Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

Image characterization
"Correlation" (1)

Characterization with correlation

	Equation
Correlation	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1}\frac{(i-m_r)(j-m_c)}{\sigma_r\sigma_c}p(i,j)$

	Equation
Row-mean and standard deviation	$m_r=\sum_{i=0}^{2^q-1} i\sum_{j=0}^{2^q-1} p(i,j)$ $\quad \sigma_r^2=\sum_{i=0}^{2^q-1}(i-m_r)^2\sum_{j=0}^{2^q-1} p(i,j)$
Column-mean and standard deviation	$m_c=\sum_{j=0}^{2^q-1} j\sum_{i=0}^{2^q-1} p(i,j)$ $\quad \sigma_c^2=\sum_{j=0}^{2^q-1}(j-m_c)^2\sum_{i=0}^{2^q-1} p(i,j)$

Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

e.g.

k (intensity levels) - j

k (intensity levels) - i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0								
1								
2								
3				1				
4								
5								
6								
7								

	values
mean	$m_{r,c}=3\times1=3$
standard deviation	$\sigma_{r,c}=(3-3)^2\times1=0$
Correlation	*Not applicable*

in a constant image, the correlation can't be defined if the standard deviation is zero

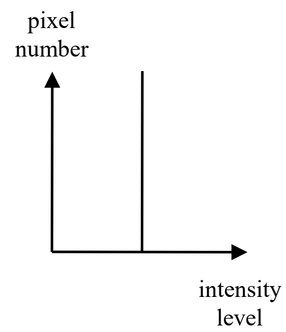


Image characterization
"Correlation" (2)

Characterization with correlation

	Equation
Correlation	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1}\frac{(i-m_r)(j-m_c)}{\sigma_r\sigma_c}p(i,j)$

	Equation
Row-mean and standard deviation	$m_r=\sum_{i=0}^{2^q-1} i\sum_{j=0}^{2^q-1}p(i,j)$ $\quad \sigma_r^2=\sum_{i=0}^{2^q-1}(i-m_r)^2\sum_{j=0}^{2^q-1}p(i,j)$
Column-mean and standard deviation	$m_c=\sum_{j=0}^{2^q-1} j\sum_{i=0}^{2^q-1}p(i,j)$ $\quad \sigma_c^2=\sum_{j=0}^{2^q-1}(j-m_c)^2\sum_{i=0}^{2^q-1}p(i,j)$

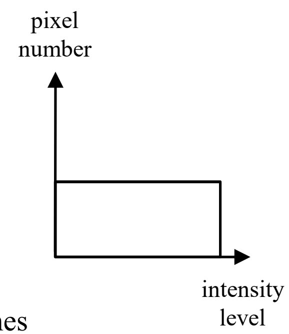
Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

e.g.

k (intensity levels) - j

k (intensity levels) - i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1

	values
mean	$m_{r,c}\rightarrow\frac{2^q-1}{2}$
standard deviation	$\sigma_{r,c}\rightarrow\infty$
Correlation	$\rightarrow 0$



in a random image,
the correlation reaches
the minimum 0

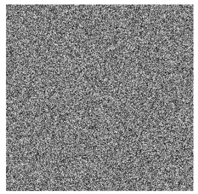


Image characterization
“Correlation” (3)

Characterization with correlation

	Equation
Correlation	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1}\frac{(i-m_r)(j-m_c)}{\sigma_r\sigma_c}p(i,j)$

	Equation
Row-mean and standard deviation	$m_r=\sum_{i=0}^{2^q-1} i\sum_{j=0}^{2^q-1}p(i,j) \quad \sigma_r^2=\sum_{i=0}^{2^q-1}(i-m_r)^2\sum_{j=0}^{2^q-1}p(i,j)$
Column-mean and standard deviation	$m_c=\sum_{j=0}^{2^q-1} j\sum_{i=0}^{2^q-1}p(i,j) \quad \sigma_c^2=\sum_{j=0}^{2^q-1}(j-m_c)^2\sum_{i=0}^{2^q-1}p(i,j)$

Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

e.g.

k (intensity levels) - j

k (intensity levels) - i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0								1
1								
2								
3								
4								
5								
6								
7	1							

	values
mean	$m_{r,c}=0\times\frac{1}{2}+7\times\frac{1}{2}=3{,}5$
standard deviation	$\sigma_{r,c}=(0-3{,}5)^2\times\frac{1}{2}+(7-3{,}5)^2\times\frac{1}{2}=3{,}5^2$
Correlation	$\frac{(0-3{,}5)\times(7-3{,}5)}{\left(\sqrt{3{,}5^2}\right)^2}\times\frac{1}{2}+\frac{(7-3{,}5)\times(0-3{,}5)}{\left(\sqrt{3{,}5^2}\right)^2}\times\frac{1}{2}$ $=-1$

in a band image with pixels that co-occur the correlation reaches the “maximum” -1

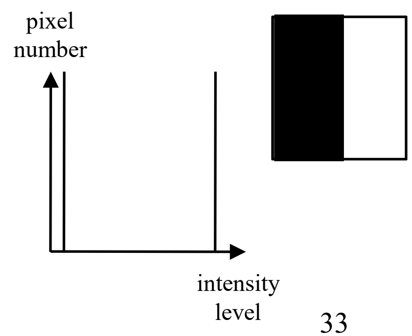


Image characterization
"Correlation" (4)

Characterization with correlation

	Equation
Correlation	$\sum_{i=0}^{2^q-1}\sum_{j=0}^{2^q-1}\frac{(i-m_r)(j-m_c)}{\sigma_r\sigma_c}p(i,j)$

	Equation
Row-mean and standard deviation	$m_r=\sum_{i=0}^{2^q-1} i\sum_{j=0}^{2^q-1}p(i,j) \quad \sigma_r^2=\sum_{i=0}^{2^q-1}(i-m_r)^2\sum_{j=0}^{2^q-1}p(i,j)$
Column-mean and standard deviation	$m_c=\sum_{j=0}^{2^q-1} j\sum_{i=0}^{2^q-1}p(i,j) \quad \sigma_c^2=\sum_{j=0}^{2^q-1}(j-m_c)^2\sum_{i=0}^{2^q-1}p(i,j)$

Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

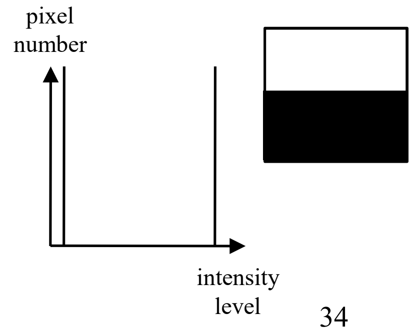
e.g.

k (intensity levels) - j

k (intensity levels) - i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	1							
1								
2								
3								
4								
5								
6								
7								1

	values
mean	$m_{r,c}=0\times\frac{1}{2}+7\times\frac{1}{2}=3,5$
standard deviation	$\sigma_{r,c}=(0-3,5)^2\times\frac{1}{2}+(7-3,5)^2\times\frac{1}{2}=3,5^2$
Correlation	$\frac{(0-3,5)^2}{\left(\sqrt{3,5^2}\right)^2}\times\frac{1}{2}+\frac{(7-3,5)^2}{\left(\sqrt{3,5^2}\right)^2}\times\frac{1}{2}=1$

in a band image with pixels that don't co-occur the correlation reaches the "maximum" 1



Histogram-based operators

1. Fundamentals of histogram-based operators
2. Some histogram-based operators
   2.1. Image characterization
   2.2. Optimum thresholding
   2.3. Histogram equalization
   2.4. Histogram-based distances
3. Further investigations

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
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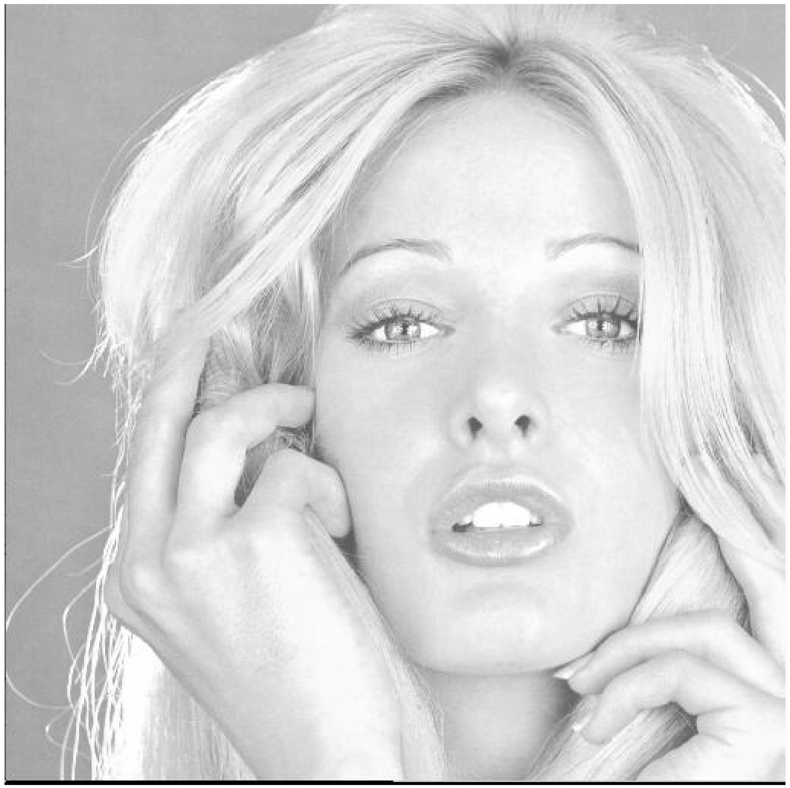
Optimum thresholding
“Otsu’s method” (1)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu’s method is optimum in the sense that it maximizes the between-class variance.

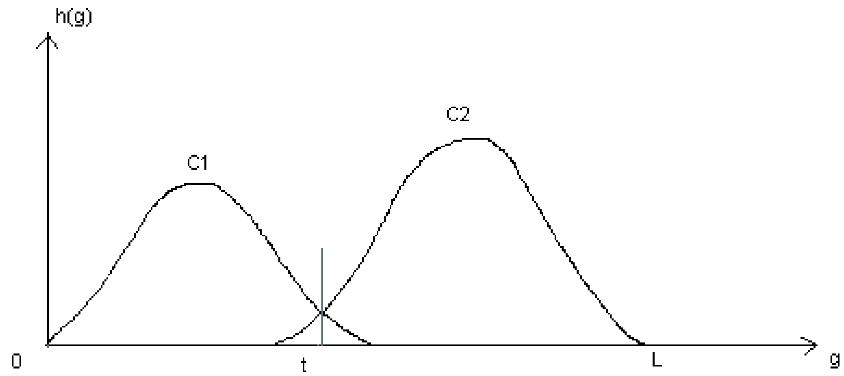


Optimum thresholding
"Otsu's method" (2)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu's method is optimum in the sense that it maximizes the between-class variance.

	Equation
Probability density function (i.e. normalized histogram)	$\sum_{k=0}^{2^q-1} p(k) = 1$
Mean	$m = \sum_{k=0}^{2^q-1} k \times p(k)$

	Equation
Probability P_i of a class C_i to have a pixel assigned to it	$P_1 = \sum_{k=0}^{t} p(k)$ $P_2 = \sum_{k=t+1}^{2^q-1} p(k)$ $P_1 + P_2 = 1$
Mean intensity value of the pixel assigned to class C_i	$m_1 = \frac{1}{P_1} \sum_{k=0}^{t} k \times p(k)$ $m_2 = \frac{1}{P_2} \sum_{k=t+1}^{2^q-1} k \times p(k)$ $P_1 \times m_1 + P_2 \times m_2 = m$

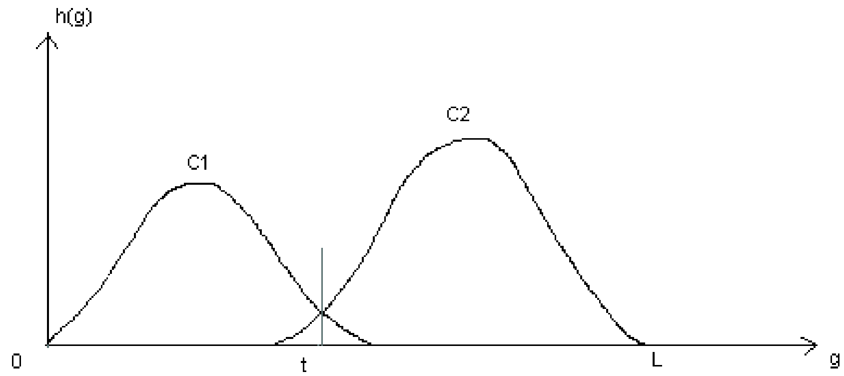


Optimum thresholding
“Otsu’s method” (3)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu’s method is optimum in the sense that it maximizes the between-class variance.

	Equation
Variance	$\sigma^2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$

	Equation
The C_i variance	$\sigma_1^2 = \frac{1}{P_1} \sum_{k=0}^{t} (k-m_1)^2 \times p(k)$ $\quad \sigma_2^2 = \frac{1}{P_2} \sum_{k=t+1}^{2^q-1} (k-m_2)^2 \times p(k)$
The between-class and intra-class variances	$\sigma_B^2 = P_1(m_1-m)^2 + P_2(m_2-m)^2$ $\quad \sigma_I^2 = P_1\sigma_1^2 + P_2\sigma_2^2$ $\sigma^2 = \sigma_B^2 + \sigma_I^2$
Goodness of the threshold	$\eta = \frac{\sigma_B^2}{\sigma^2}$



Optimum thresholding
"Otsu's method" (4)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu's method is optimum in the sense that it maximizes the between-class variance.

e.g.

k	0	1	2	3	4	5	6	7
h(k)	40	30	20	10	20	30	40	50
p(k)	4/24	3/24	2/24	1/24	2/24	3/24	4/24	5/24
P_1(k)	4/24	7/24	9/24	10/24	12/24	15/24	19/24	Na
P_2 (k)	20/24	17/24	15/24	14/24	12/24	9/24	5/24	Na
m_1 (k)	0	0.42	0.77	1	1.5	2.2	3	Na
m_2 (k)	4.60	5.23	5.66	5.85	6.16	6.55	7	Na
σ^2_1 (k)	0	0.24	0.61	1	2.08	3.62	5.26	Na
σ^2_2 (k)	4.64	2.76	1.55	1.12	0.63	0.24	0	Na
σ^2_B (k)	2.93	4.77	5.6	5.73	5.44	4.44	2.63	Na
σ^2_I (k)	3.86	2.03	1.20	1.07	1.36	2.35	4.16	Na
η(k)	0.43	0.70	0.82	**0.84**	0.80	0.65	0.38	Na

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 4 \qquad \sigma^2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k) = 6.8$$

$$P_1 = \sum_{k=0}^{t} p(k) \qquad P_2 = \sum_{k=t+1}^{2^q-1} p(k) \qquad P_1 + P_2 = 1$$

$$m_1 = \frac{1}{P_1}\sum_{k=0}^{t} k \times p(k) \qquad m_2 = \frac{1}{P_2}\sum_{k=t+1}^{2^q-1} k \times p(k) \qquad P_1 \times m_1 + P_2 \times m_2 = m$$

$$\sigma_1^2 = \frac{1}{P_1}\sum_{k=0}^{t} (k-m_1)^2 \times p(k) \qquad \sigma_2^2 = \frac{1}{P_2}\sum_{k=t+1}^{2^q-1} (k-m_2)^2 \times p(k)$$

$$\sigma_B^2 = P_1(m_1-m)^2 + P_2(m_2-m)^2 \qquad \sigma_I^2 = P_1\sigma_1^2 + P_2\sigma_2^2 \qquad \sigma^2 = \sigma_B^2 + \sigma_I^2$$

$$\eta = \frac{\sigma_B^2}{\sigma^2}$$

Optimum thresholding
"Otsu's method" (5)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu's method is optimum in the sense that it maximizes the between-class variance.

e.g. Implementation in Matlab of the Otsu algorithm

```
% probability density
k = [0 1 2 3 4 5 6 7];
h = [40 30 20 10 20 30 40 50];
p = h / sum(h); sum(p);

% probability
p1 = zeros(1,7); p2 = zeros(1,7);
for t=1:7
    p1(t) = sum(p(1:t));
    p2(t) = sum(p(t+1:8));
end
p1 + p2;

% global mean
m = sum(k.*p);

% mean
m1 = zeros(1,7); m2 = zeros(1,7);
for t=1:7
    m1(t) = sum(k(1:t).*p(1:t)) / p1(t);
    m2(t) = sum(k(t+1:8).*p(t+1:8)) / p2(t);
end
(p1.*m1 + p2.*m2)-m;
```

```
% global variance
v = sum( ((k-m).^2) .*p  );

% variance
v1 = zeros(1,7); v2 = zeros(1,7);
for t=1:7
    v1(t) =  sum ( ((k(1:t)-m1(t)).^2).*p(1:t) ) / p1(t);
    v2(t) =  sum ( ((k(t+1:8)-m2(t)).^2).*p(t+1:8) ) / p2(t
end

% i and b variances
vb = zeros(1,7); vi = zeros(1,7);
for t=1:7
    vb(t) = p1(t)*(m1(t)-m)^2 + p2(t)*(m2(t)-m)^2;
    vi(t) = p1(t)*v1(t) + p2(t)*v2(t);
end
(vi+vb)-v;

% goodness
g = zeros(1,7); g = vb / v; [Y,I] = max(g);
```

Histogram-based operators

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   2.2. Optimum thresholding
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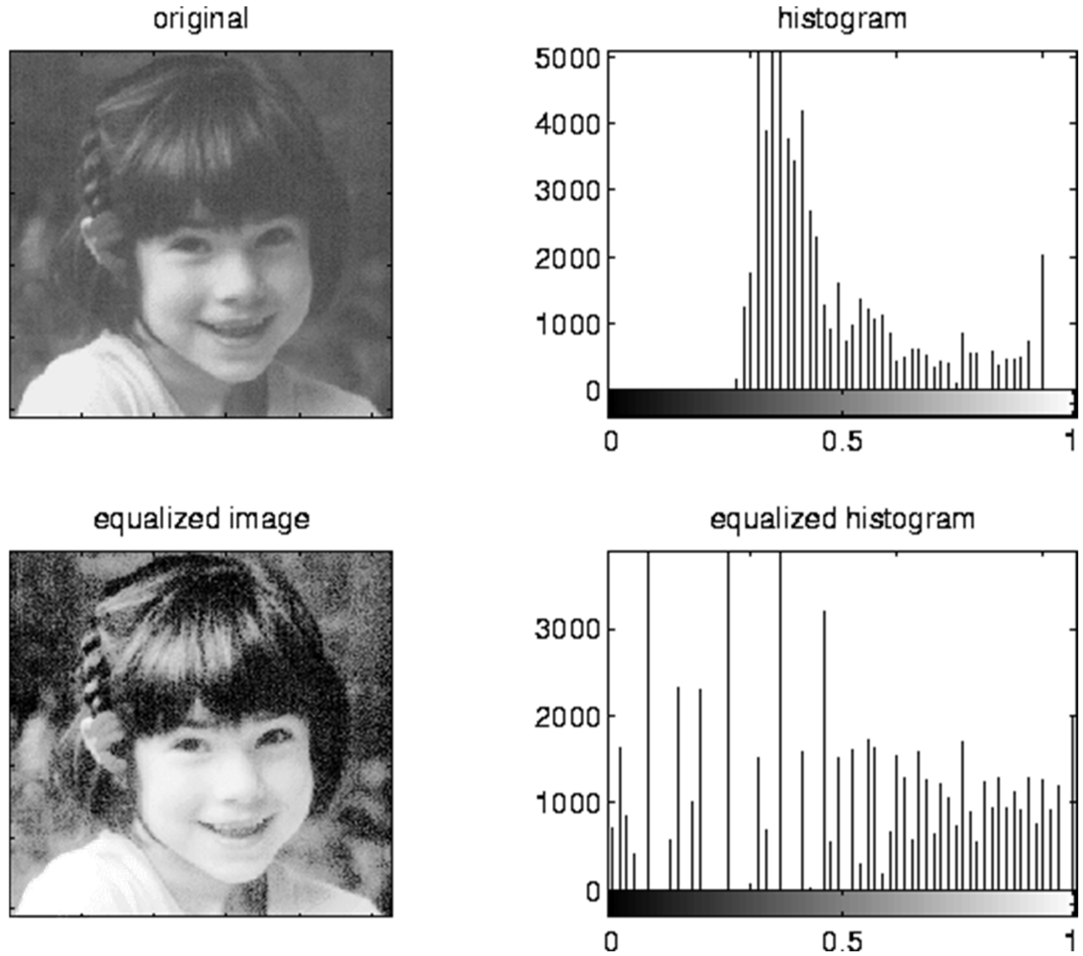
   2.4. Histogram-based distances
3. Further investigations

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

Histogram equalization (1)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.



Histogram equalization (2)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram *r*	$\sum_{k=0}^{2^q-1} r(k)$
The output (equalized) histogram *s*	$\sum_{k=0}^{2^q-1} s(k)$

	Equation	Comments
Size equality	$\sum_{k=0}^{2^q-1} r(k) = \sum_{k=0}^{2^q-1} s(k) = NM$	Since the size is preserved in the input and output images, we have
Mapping rule for cumulative histograms *r(k∈[0,u]), s(k∈[0,v])*	$\sum_{k=0}^{u} r(k) = \sum_{k=0}^{v} s(k)$	The cumulative input histogram *r* up to level *u* should be transformed to cover up to level *v* in the output cumulative histogram *s*
Uniform output discrete function s(k)	$s(k) = \frac{NM}{2^q - 1} \quad \forall k$	Since the output histogram *s* is uniformly flat, we have
Uniform cumulative histograms *r(k∈[0,u]), s(k∈[0,v])*	$\sum_{k=0}^{v} s(k) = v \times \frac{NM}{2^q - 1} = \sum_{k=0}^{u} r(k)$	The cumulative histograms *r,s* of output and input images are then equal to
Mapping function	$v = \frac{(2^q - 1)}{NM} \sum_{k=0}^{u} r(k) = (2^q - 1) \sum_{k=0}^{u} p_r(k)$ $with \quad p_r(k) = \frac{r(k)}{NM}$ $v = T(u) = (2^q - 1) \sum_{k=0}^{u} p_r(k)$	The mapping function, for a pixel at level *v*, from the input pixel at level *u*, is then

Histogram equalization (3)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram *r*	$\sum_{k=0}^{2^q-1} r(k)$
Mapping function	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1)\sum_{k=0}^{u} p_r(k)$
The output (equalized) histogram *s*	$\sum_{k=0}^{2^q-1} s(k)$

Histogram equalization (4)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram *r*	$\sum_{k=0}^{2^q-1} r(k)$
Mapping function	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1)\sum_{k=0}^{u} p_r(k)$
The output (equalized) histogram *s*	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following *r* histogram
(1) we compute p_r first

k	***r(k)***	***$p_r(k)$***
0	790	0,19
1	1023	0,25
2	850	0,21
3	656	0,16
4	329	0,08
5	245	0,06
6	122	0,03
7	81	0,02

$$\sum_{k=0}^{2^q-1} r(k) = 4096 = 64^2$$

Histogram equalization (5)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram *r*	$\sum_{k=0}^{2^q-1} r(k)$
Mapping function	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1)\sum_{k=0}^{u} p_r(k)$
The output (equalized) histogram *s*	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following *r* histogram (2) when p_r known, we compute T(u)

k	***p_r(k)***
0	0,19
1	0,25
2	0,21
3	0,16
4	0,08
5	0,06
6	0,03
7	0,02

u	***T(u)***
0	1,33
1	3,08
2	4,55
3	5,67
4	6,23
5	6,65
6	6,86
7	7

$$= T(2) = 7\sum_{k=0}^{2} p_r(k) = 7 \times (0{,}19 + 0{,}25 + 0{,}21)$$

$$= T(5) = 7\sum_{k=0}^{5} p_r(k)$$

$$= 7 \times (0{,}19 + 0{,}25 + 0{,}21 + 0{,}16 + 0{,}08 + 0{,}06)$$

Histogram equalization (6)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram *r*	$\sum_{k=0}^{2^q-1} r(k)$
Mapping function	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1)\sum_{k=0}^{u} p_r(k)$
The output (equalized) histogram *s*	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following *r* histogram
(3) T(u) have fractions because they are generated by summing probabilities, so T'(u) round them to the nearest integers

k	***$p_r(k)$***
0	0,19
1	0,25
2	0,21
3	0,16
4	0,08
5	0,06
6	0,03
7	0,02

u	***T(u)***	***T'(u)***
0	1,33	1
1	3,08	3
2	4,55	5
3	5,67	6
4	6,23	6
5	6,65	7
6	6,86	7
7	7	7

Histogram equalization (7)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram *r*	$\sum_{k=0}^{2^q-1} r(k)$
Mapping function	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1)\sum_{k=0}^{u} p_r(k)$
The output (equalized) histogram *s*	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following *r* histogram
(4) when T'(u) known, we can obtain s

k	***r(k)***
0	790
1	1023
2	850
3	656
4	329
5	245
6	122
7	81

u	***T'(u)***
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

k	***s(k)***	
0	0	
1	790	$= r(0)$
2	0	
3	1023	$= r(1)$
4	0	
5	850	$= r(2)$
6	985	$= r(3) + r(4) = 656 + 329$
7	448	$= r(5) + r(6) + r(7) = 245 + 122 + 81$

Histogram equalization (8)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram *r*	$\sum_{k=0}^{2^q-1} r(k)$
Mapping function	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1)\sum_{k=0}^{u} p_r(k)$
The output (equalized) histogram *s*	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following *r* histogram (5) and then p_s

k	***r(k)***	***p_r(k)***
0	790	0,19
1	1023	0,25
2	850	0,21
3	656	0,16
4	329	0,08
5	245	0,06
6	122	0,03
7	81	0,02

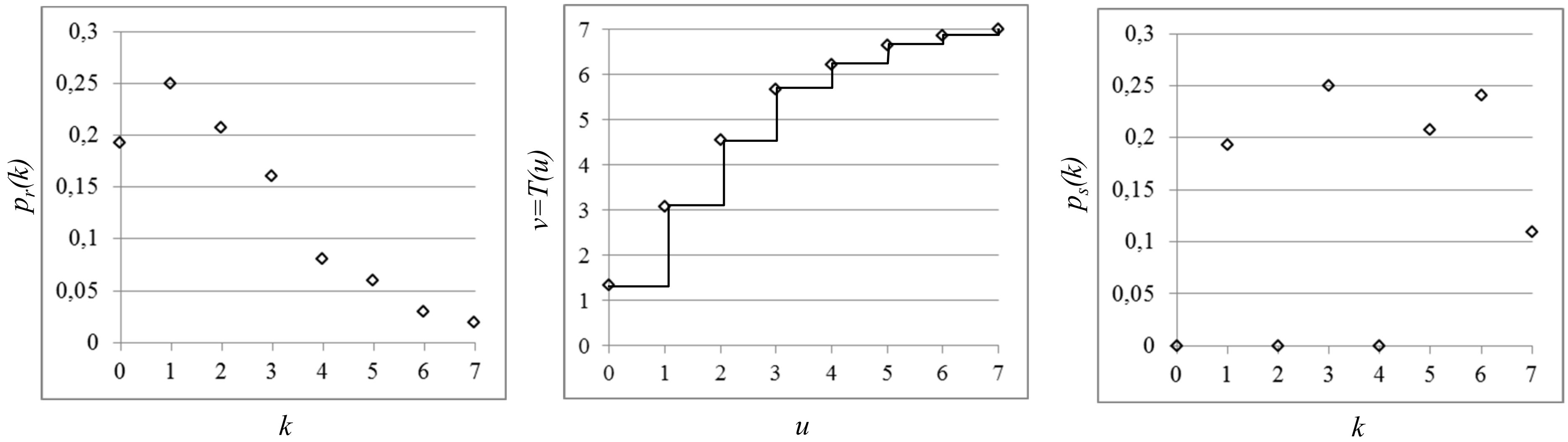
u	***T'(u)***
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

k	***s(k)***	***p_s(k)***
0	0	0
1	790	0,19
2	0	0
3	1023	0,25
4	0	0
5	850	0,21
6	985	0,24
7	448	0,11

Histogram equalization (9)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

e.g. the final plot



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Histogram-based distance	Minkowski, χ^2, Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

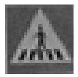
Histogram-based distances (1)

Histogram-based distances: given two (sub)images A and B whose (normalized) histograms are h_A and h_B, we define an histogram-based distance as $D(h_A, h_B)$ between the histograms h_A and h_B.

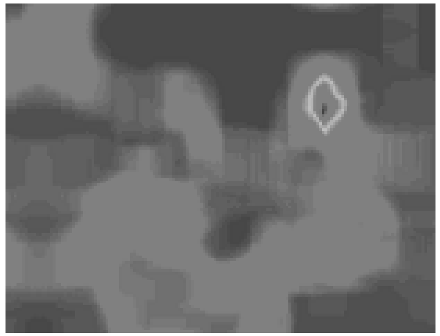
Example Usage:

- Tracking
- Image retrieval
- Registration
- Detection
- Many more ...

input



target



similarity

Histogram-based distances (2)

Histogram-based distances: given two (sub)images A and B whose (normalized) histograms are h_A and h_B, we define an histogram-based distance as $D(h_A, h_B)$ between the histograms h_A and h_B.

e.g. within bin-by-bin distances, only pairs of bins in the two histograms that have the same index are matched.

Minkowski distance: is a metric on Euclidean space which can be considered as a generalization of both the Euclidean distance and the Manhattan distance.

$$D_p(A,B) = \left[\sum_{k=0}^{2^q-1} \left| h_A(k) - h_B(k) \right|^p \right]^{1/p}$$

p	D(A,B)
-∝	min distance
1	Manhattan distance
2	Euclidean distance
+∝	max distance

χ^2 statistics: measures how unlikely it is that one distribution was drawn from the population represented by the other.

$$D_{\chi^2}(A,B) = \sum_{k=0}^{2^q-1} \frac{(h_A(k) - m(k))^2}{m(k)}$$

$$m(k) = \frac{h_A(k) + h_B(k)}{2}$$

Histogram-based distances (3)

Histogram-based distances: given two (sub)images A and B whose (normalized) histograms are h_A and h_B, we define an histogram-based distance as $D(h_A, h_B)$ between the histograms h_A and h_B.

e.g. within bin-by-bin distances, only pairs of bins in the two histograms that have the same index are matched.

Kullback-Leibler (KL) divergence measures the amount of added information needed to encode image A based on the histogram of image B. The KL divergence is not symmetric $D_{KL}(A,B) \neq D_{KL}(B,A)$, and then not a distance

$$D_{KL}(A,B) = -\sum_{k=0}^{2^q-1} h_A(k) \log \frac{h_A(k)}{h_B(k)}$$

Jeffrey distance: is a modification of the K-L divergence that is numerically stable, symmetric (then a distance) and robust with respect to noise and the size of histogram bins

$$D_J(A,B) = \sum_{k=0}^{2^q-1} h_A(k) \log \frac{h_A(k)}{m(k)} + h_B(k) \log \frac{h_B(k)}{m(k)}$$

$$m(k) = \frac{h_A(k) + h_B(k)}{2}$$

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Further investigations

- Other characterization methods / features (e.g. the Intensity Variation Number)
- Multiple thresholding (extension of the Otsu method)
- Histogram specification
- Comparison of histograms (cross-bin measures)
- Etc.